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An investigation of cosmic rays underground using a scintillator stack

V. A study of groups of muons penetrating underground

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Abstract. The scintillator stack was operated for eight months in coincidence with two additional counters placed in the same tunnel at distances up to 6·3 m away. It is shown that this arrangement is satisfactory for studying muon showers and that the stack alone can be used to distinguish groups of muons. The measured rate of pairs of muons is compared with results from other experiments at various depths. The results are compatible with Greisen's expression for the muon density in air showers; in the extrapolated region close to the shower axis it is necessary to assume that the muon density rises more sharply than at greater distances, but there is no evidence for very strongly collimated groups of muons.

1. Introduction

The main emphasis in experiments on extensive cosmic ray showers has slowly changed from studying the electron–photon component to studying the muon component, as it has become clear that the latter carries a larger share of the original energy. It is usually difficult to make a clear distinction between the various components at sea level, so there is an obvious advantage in observing the muons underground. The most effective studies are usually those in which observations are made simultaneously at sea level and underground, as in the classic experiment of Barrett *et al.* (1952), but there are some aspects of the muon shower phenomena which can usefully be investigated in a purely underground experiment.

The work reported in this paper describes observations made with the scintillator stack when it was operated in coincidence with other counters placed a short distance away from it in the same laboratory, at a depth of 60 m.w.e. below ground level. It will be shown that the majority of the observed coincidences were due to two or more muons traversing the counters simultaneously.

A short account of part of these investigations was presented at the I.U.P.A.P. London conference on cosmic rays in September 1965 (Barton 1965).

2. Method of observation

The scintillator stack S, consisting of six 1·12 m^2 counters interleaved with 1·3 cm layers of lead, was operated in the same manner as described by Barton (1966 a), to be referred to as I. Two additional counters X_1 and X_2, exactly similar to those in the stack, were arranged as shown in figure 1. The nearer counter was mounted on rails so that it could readily be operated at different distances *d* from the stack; the other counter was at a fixed distance of 6·3 m. (The distances were measured between the centres of the counters.)

Recorded events were those in which there was a coincidence between S and either X_1 or X_2. The resolving time was 150 ns, which ensured that accidental coincidences were less than 2% of the observed rates. In a proportion of the events both X_1 and X_2 were discharged; these $S+X_1+X_2$ events will be referred to as threefold coincidences. (It is to be noted that they are also included in the totals of twofold coincidences.) The pulse heights from the X counters were encoded and recorded together with the data from the stack.

The analysis and classification of the events was similar to that described in I. The proportion of incomplete events, in which not all of the stack counters recorded a particle, was several times larger than when the stack was operated on its own. This was not unexpected as the earlier studies had indicated that most of the incomplete events could be accounted for by showers striking the sides of the stack. No attempt has been made to analyse this type of event and the data in this paper refer only to events in which all the stack counters recorded the passage of at least one particle.

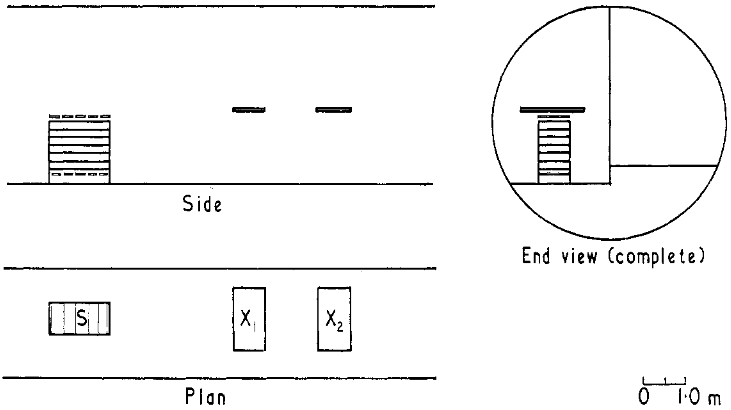


Figure 1. General arrangement.

3. First series of experiments

These results, which are presented in the top part of table 1, were obtained without lead above either X_1 or X_2. A distinction has been made between those events in which the pulse height from the top counter of the stack indicated that only a single particle was incident on it and those involving more particles. For the twofold coincidences both types of event increased rapidly as the separation was reduced but the effect was larger for the more complicated events. The threefold events, which always required a particle through X_2 at 6·3 m, were not appreciably dependent on the position of the intermediate counter; about 60% had more than one particle incident on the stack. These results suggested that at least two distinct physical processes were involved. Examination of the stack index counter records showed that for all threefold coincidences and for the twofold coincidences at separations of greater than 4 m the directions of particles traversing the stack were distributed symmetrically. However, for twofold coincidences at closer separations there was a great excess of particles with inclinations such that they had emerged from the tunnel roof on the same side as the X_1 counter. Owing to the geometry of the arrangement, this is the direction from which muons accompanied by knock-on showers from the rock had the greatest probability of giving a coincidence, and it is therefore likely that the excess events were due to this effect.

4. Second series of experiments

For the remainder of this investigation the X counters were covered with a 5 cm layer of lead which overhung the area of the counters by a sufficient margin to ensure that any particle within 45° of the vertical would pass through it. The results of these runs, given in the lower part of table 1, show that the lead absorbed a large part of the increased rates for twofold coincidences at closer separations. The index counters no longer showed any asymmetry and thus confirmed that most of the soft knock-on showers had been eliminated. The pulse-height distributions from the X counters showed that they were recording particles which traversed the full thickness of scintillator so, unlike the case with Geiger counters, 5 cm of lead appeared to be sufficient to reject the soft component. This suggests that the remaining variation with separation was due to some other reason.

In addition to the runs reported in table 1, the apparatus was operated for a further period of 130 days with the stack running under the 'bias 1' condition, described in I, which required that at least two particles traversed it. The data from this run were used solely to improve the statistics of the rarer types of event and will be included when necessary in the following analysis.

Table 1. Data summary

d (m)	Running time (d)	Total number of complete events	Rates of twofold coincidences S + X		Rates of threefold coincidences S + X_1 + X_2	
Results without lead over X_1 and X_2			One particle incident on stack (d^{-1})	More than one particle incident on stack (d^{-1})	One particle incident on stack (d^{-1})	More than one particle incident on stack (d^{-1})
2·1	6·1	1707	146 ± 5	133 ± 5	8·1 ± 1·1	9·9 ± 1·3
2·7	7·0	1192	91 ± 3	79 ± 3	9·1 ± 1·1	10·9 ± 1·3
3·3	11·5	1356	69 ± 3	49 ± 2	6·9 ± 0·8	12·4 ± 1·0
3·9	19·3	1858	62 ± 2	35 ± 2	8·9 ± 0·7	12·5 ± 0·8
4·5	16·9	1465	58 ± 2	29 ± 2	8·7 ± 0·7	12·2 ± 0·8
5·1	19·9	1410	48 ± 2	23 ± 1	8·5 ± 0·6	12·0 ± 0·8
6·3	80·7	5331	50 ± 1	18 ± 1	—	—
Results with 5 cm lead above X_1 and X_2						
2·7	101·8	9494	59 ± 1	34 ± 1	6·9 ± 0·3	8·6 ± 0·3
4·5	50·6	3322	47 ± 1	19 ± 1	6·8 ± 0·4	9·1 ± 0·4
6·3	152·4	10028	50 ± 1	15 ± 1	—	—

5. Analysis of results

The most striking feature of the results is the proportion of events in which the stack was traversed by more than one particle. This is brought out more clearly in the first column of table 2, which compares the results for the stack operated on its own with those obtained when in coincidence with the lead-covered counter at 6·3 m distance. The events in which a single particle enters the stack are also interesting because, as shown in the second line of the table, their probability of interacting in the stack is higher than for random muons. This latter result has already been discussed (Barton 1965) and shown to imply a much higher average energy for muons arriving underground in showers than for unaccompanied ones. The data on which this conclusion was reached have been re-examined, but since this has only served to strengthen the previously stated view nothing further will be reported here on this topic.

Table 2. Proportions of multiple events and of interacting particles

	(1)	(2)
Stack operated on its own	3·0 ±0·1%	12·1 ±0·2%
Stack operated in coincidence with lead-covered counter at 6·3 m	22·9 ±1·0%	21·6 ±1·2%

(1) Proportion of all events in which more than one particle is incident on the stack;
(2) proportion of singly incident particles which interact in the stack.

In previous papers (Barton *et al.* 1966, Barton 1966 b, to be referred to as II and III, respectively), it was shown that the rates for 'interactions' in the stack were in agreement with the expected cross sections for electromagnetic processes due to muons. It would be expected that those muons interacting in the rock would emerge together with an electromagnetic cascade and be recorded as 'incident showers'; no detailed analysis was made of these events, since the locations of the original interactions could not be deduced unambiguously, but their general properties were consistent with those expected.

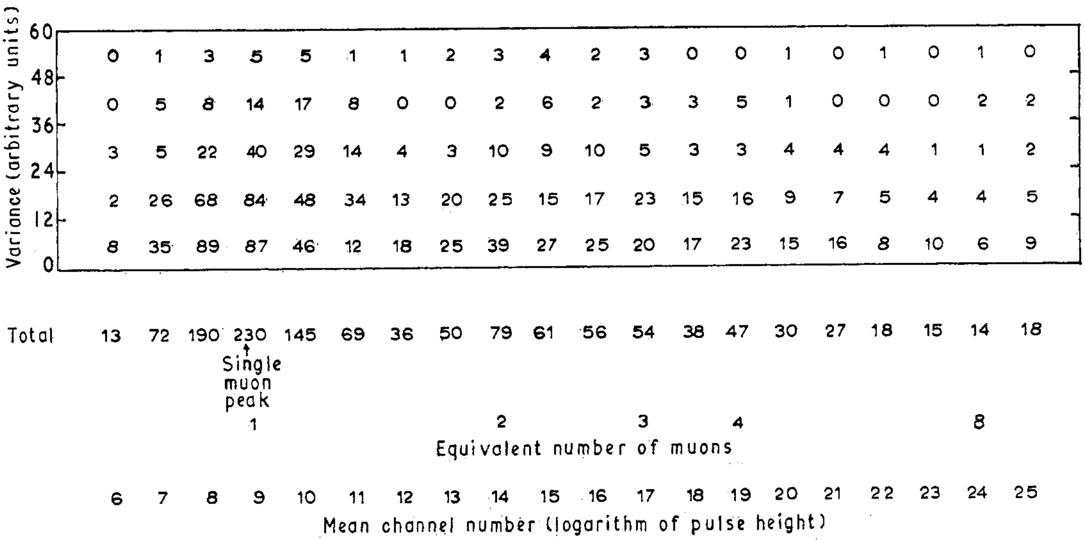
For the shower-triggered events discussed in this paper there should also be a proportion of events in which the muon traversing the stack is accompanied by an electromagnetic shower from the rock. Since the muons were observed to have a higher interaction probability there should be more of these events. However, the increase in the interaction rate will be quite small in rock—direct pair production is largely responsible for the increase in the stack interactions and this is a Z^2-dependent process. Since, also, shower penetration only increases logarithmically with energy it follows that the proportion of extensive shower-triggered events having more than one particle entering the stack should not be very much greater than the 3% observed for all stack events. It is therefore concluded that most of the 23% recorded in table 2 must have been due to some other process. Only one set of data is given in table 2 but observations with X at different distances gave rather similar results, with the proportion increasing to 36% for X at 2·7 m. For the threefold coincidences involving both X_1 and X_2 the proportion of events with more than one particle incident on the stack was 55%.

If it is accepted that most of the events discussed in the previous paragraph were not due to soft showers accompanying muons, then they must have been due either to penetrating showers produced in the rock or to multiple muons coming from the atmosphere. The first of these possibilities may be important for the smaller separations but can hardly account for the events observed with 6·3 m separation, because the tunnel roof was only an average of 2·5 m above the counters and high-energy showers would be strongly collimated. The relatively small decrease on adding 5 cm of lead above the X_2 counter reinforces the view that locally produced showers were not responsible for a major part of the observed events. It is therefore concluded that, unless some quite new phenomenon is involved, the observations must be explained as due to groups of muons generated in the atmosphere.

6. Analysis of muon groups

The above argument showed that coincident events often involved more than one muon falling on each counter. With single scintillation counters it is never possible to distinguish whether a particular large pulse is due to a muon accompanied by a soft shower or to a number of muons. In the case of a stack of counters the distinction can be made, at least in principle, by observing whether the relative pulse heights show signs of electromagnetic shower development. An attempt has therefore been made to devise a computer technique which does this.

The method adopted was to find the mean of the six pulse heights from the stack counters and then compute the variance of the six pulse heights about this mean. This was repeated for each event and a two-dimensional distribution of the events constructed with mean pulse height and variance as the independent variables. An example of this analysis is given in figure 2, which shows the plot for all threefold events $S+X_1+X_2$. In the computing process events which were clearly accompanied by electromagnetic shower development were dropped from the analysis; otherwise the proportion of events with very high internal variance would be much larger than in figure 2. It is seen from this figure that events in which two or more muons traverse the stack can be distinguished from those involving only a single muon. The resolution is poor if all events are included, but quite good if events of high variance are rejected on the grounds that these events can be explained as due to muons interacting in the stack.



Variance (arbitrary units)	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
48–60	0	1	3	5	5	1	1	2	3	4	2	3	0	0	1	0	1	0	1	0
36–48	0	5	8	14	17	8	0	0	2	6	2	3	3	5	1	0	0	0	2	2
24–36	3	5	22	40	29	14	4	3	10	9	10	5	3	3	4	4	4	1	1	2
12–24	2	26	68	84	48	34	13	20	25	15	17	23	15	16	9	7	5	4	4	5
0–12	8	35	89	87	46	12	18	25	39	27	25	20	17	23	15	16	8	10	6	9
Total	13	72	190	230	145	69	36	50	79	61	56	54	38	47	30	27	18	15	14	18

Figure 2. Mean–variance plot for $S+X_1+X_2$ events.

Similar plots for twofold events $S+X$ were also computed and found to show rather poorer resolution. For the stack operated alone it was quite impossible to distinguish the double muon peak, as the very high ratio of single muons to muon showers made the experimental conditions very unfavourable; this was true even if events were selected by requiring at least two index counters in each tray to be discharged. The data are plotted on figure 3, in which only events in the lowest variance group are included.

An important result from figure 3 is that although the shower-triggered events were less than 10^{-3} of all the stack events there was very little difference between the rates at high muon multiplicities. This means that dense showers of muons underground are not collimated strongly into narrow bundles but extend for distances of at least several metres.

The pulse heights from the X_1 and X_2 counters have also been studied but have not yielded much additional information. Even under the most favourable conditions—covered with lead and in threefold coincidence—the double muon peak was hardly resolved (figure 4).

In general there was a correlation between the pulse heights from the X counters and those from the stack, but it is difficult to express this in a useful form. Some idea of the correlation is indicated by the fact that half of those events in which the stack registered a group of ten or more muons also showed a pulse height from one or both of the X counters which could be due to ten or more muons.

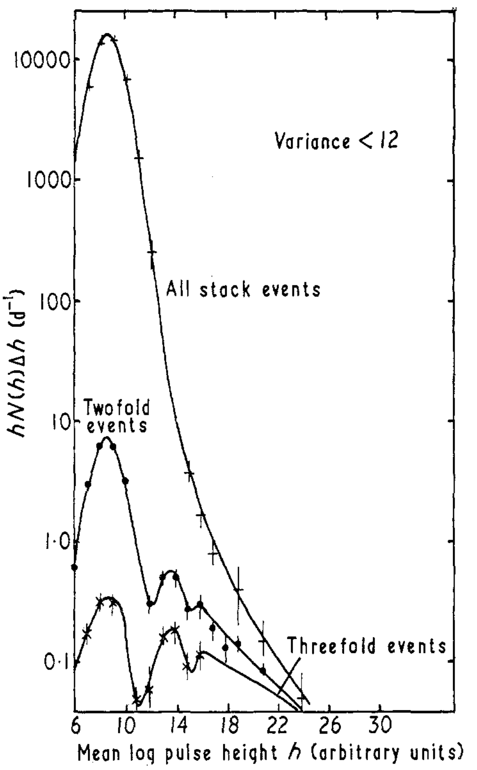


Figure 3. Distributions of mean pulse heights in stack for events of low variance.

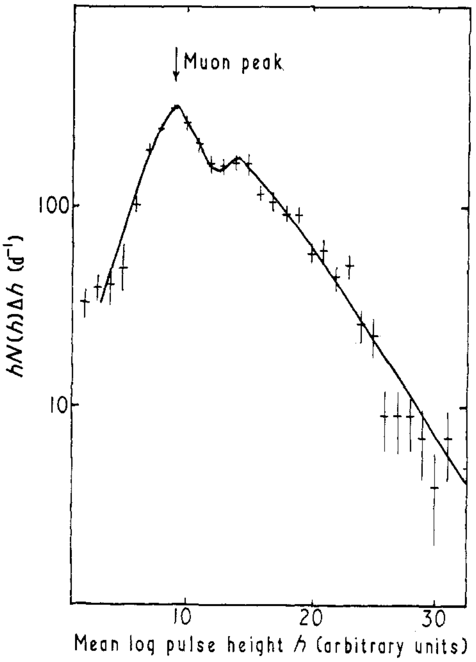


Figure 4. Pulse-height distribution in X counters.

Selecting events of lowest variance, as was done for figure 3, was useful for ensuring that only multiple muon events were included but is certainly too restrictive for estimating the absolute number of such events. It is seen in figure 2 that the pulse-height variance of single muons can be quite large. This is due partly to instrumental effects, including the statistical fluctuations in the number of photoelectrons released in the photomultipliers, and partly to low-energy knock-on interactions of the muons in the stack. Both effects will occur also for the multiple muon events, though the statistical fluctuations in the number of photoelectrons should be relatively less. For an estimate of absolute rates the three lowest variance groups of figure 2 and similar plots have been used. Although somewhat arbitrary, the uncertainty introduced would appear, from figure 2, to be only 10%.

Figure 5 shows the resulting estimate of the differential muon multiplicity spectrum for the particles traversing the stack in coincidence with the X counters. The spectrum can be fitted, for the twofold events, by a power law of the form $m^{-2\cdot6}$ for multiplicities from 5 to 100. The total rate of muon groups through the stack must be greater than those recorded in coincidence but, as discussed above, the difference will be small for the larger muon groups. For groups of less than five muons the index of the multiplicity spectrum must be larger than that given above. The total number of muon groups traversing the stack must be greater than $10 \pm 2\ \mathrm{d}^{-1}$ and the number with five or more muons in the group $1\cdot5 \pm 0\cdot3\ \mathrm{d}^{-1}$.

The rate of groups of two or more muons can be estimated independently from the S+X coincidence rates; table 1 shows that the total rate at larger separations was approximately constant at 65 d^{-1}. If it is assumed that the rate of pure muon events is appreciably independent of separation at small separations, then the rate of pairs in a single counter would be just half of this. However, the X counter is a single tray whereas each particle through the stack must be within its acceptance angle. The further reduction of rate depends slightly on the angular distribution of the events but is within the range 2·3 ± 0·2. So the rate estimated in this way is $65/2 \times 2{\cdot}3 = 14 \pm 2$ d^{-1}, in fair agreement with that deduced above. A value of 14 ± 3 d^{-1} will be adopted in the subsequent discussion.

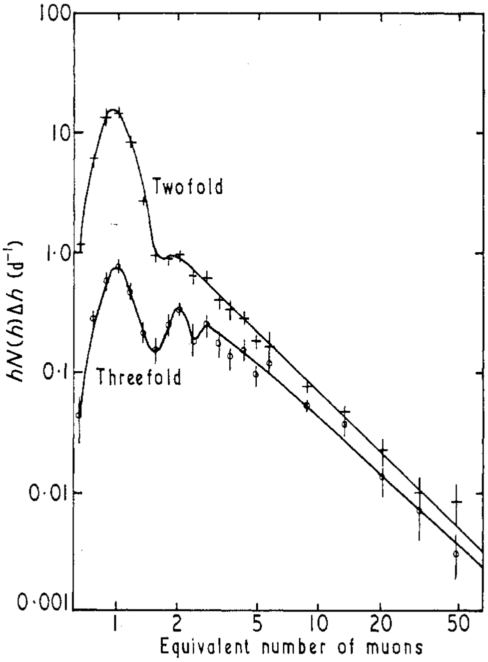


Figure 5. Final muon multiplicity distributions.

7. Comparison with other experiments

The presence of narrow groups of muons has been studied by various techniques since the original observation of Braddick and Hensby (1939). The reported data have been collected in table 3; where appropriate, the results were re-interpreted so that they could be presented in a standard form.

There are considerable difficulties in trying to compare the estimates for the rates of muon groups obtained using different apparatus. Firstly, different experimenters may choose different criteria for accepting and classifying groups in which one or more of the muons pass through the side of the apparatus. For conformity, all such events should be discarded. Secondly, the dependence of the rate on the area of the apparatus depends on the structure function of the groups. None of the results is sufficiently detailed for this to be known so it is necessary to make some provisional assumption before attempting to normalize the rates. Thirdly, the various experimental arrangements all observe the muons over a vertical path length which is not small relative to the horizontal dimensions of the apparatus. This means that the rates will also depend on the angular distribution of the muon groups, for which the only information reported (Higashi 1962) is a cos θ exponent of 4 ± 1; it is of course to be expected that the muons will be less strongly collimated than the electron–photon component of air showers so this figure appears reasonable.

It has seemed best to define the intensity of muon pairs as the number of pairs per steradian crossing a thin horizontal lamina of area one square metre in a vertical direction.

Table 3. Survey of results on multiple penetrating particles

Observers	Location (m.w.e.)	Type of apparatus	Area (m^2)	Track length (m)	Minimum number of muons	Observed rate (d^{-1})	Vertical intensity of muon pairs ($m^{-2} sr^{-1} d^{-1}$)	Notes
Barrett *et al.* 1952	1570	G.M.	0·9 × 0·9	0·6	2	0·24 ± 0·04	0·98 ± 0·16	Hodoscoped
George *et al.* 1953	60	G.M.	2 × (0·14 × 0·56)	0·25	2	2 × (1·2 ± 0·1)	260 ± 20	Local showers not excluded
Higashi *et al.* 1957	40	W.C.C.	0·8 × 0·4	0·70	4	0·3 ± 0·2	—	
Kessler and Maze 1957	65	W.C.C.	0·5 × 0·3	0·82	2	0·06 ± 0·02	32 ± 11	Some muons escaped from sides
Hunter and Trent 1962	37	G.M.	2 × (0·21 × 0·56)	0·25	2	2 × (1·17 ± 0·1)	94 ± 8	
	60					2 × (0·95 ± 0·10)	76 ± 8	
Vernov *et al.* 1962 a, b	40	G.M.	16 × (0·55 × 0·72 × ½)	0·48	?	0·17 ± 0·04	—	
Chaudhuri and Sinha 1963	148	W.C.C.	0·75 × 0·30	0·85	2	0·11 ± 0·02	21 ± 4	
Vavilov *et al.* 1963	S.L.	W.C.C.	0·60 × 0·40	0·60	2	0·6 ± 0·1	56 ± 9	
Bibilashvili *et al.* 1965	200	Spark calorimeter	0·60 × 0·50	1·8	2	0·15 ± 0·04	44 ± 7	
Creed *et al.* 1965	820	Flash tubes	1·05 × 1·00	0·42	2	3·5 ± 1·8	6·1 ± 3·0	Local showers may not all be excluded
	1810					0·6 ± 0·2	1·0 ± 0·3	
	4100					0·01	0·02	(1 event)
Bingham and Kellermann 1965	S.L. (+50 cm Pb)	G.M.	0·60 × 0·32	1·00	2	0·22 ± 0·05	83 ± 19	
Present work	60	Scintillator stack	1·50 × 0·75	1·15	2	14 ± 3	62 ± 14	

Each experimental result was then used to estimate this rate, at the relevant depth, by calculating the effective aperture *for pairs* of each apparatus. This calculation was a modification of the usual one for determining the aperture of a cosmic ray telescope. It was necessary to assume that the muon density was constant over the area of the apparatus, which is a reasonable assumption only in so far as the muons are part of air showers, and that the $\cos^4 \theta$ distribution applied at all depths, whereas of course it would be expected to vary. However, the interpretation is not critically dependent on these assumptions; for example, assuming a $\cos^8 \theta$ distribution would only increase the estimates of vertical intensity by from 5 to 25%, according to the shape of the apparatus used. The resulting values for the rate of muon pairs are given in the last column of the table and plotted in figure 6; the results of Higashi *et al.* (1957) and of Vernov *et al.* (1962 a, b) only refer to larger groups and therefore are not included.

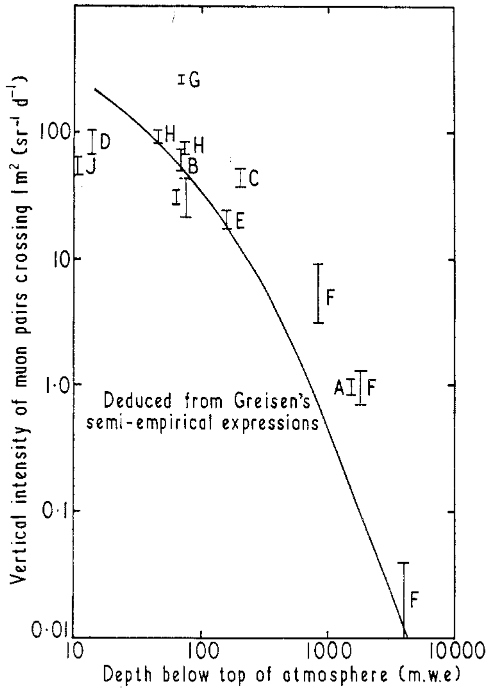


Figure 6. Vertical intensity of muon pairs below sea level: A, Barrett *et al.* 1952; B, present work; C, Bibilashvili *et al.* 1965; D, Bingham and Kellermann 1965; E, Chaudhuri and Sinha 1963; F, Creed *et al.* 1965; G, George *et al.* 1953; H, Hunter and Trent 1962; I, Kessler and Maze 1957; J, Vavilov *et al.* 1963.

If it is accepted that the remaining discrepancies between the data of figure 6 are due to the difference in technique and the difficulties in assessing the effective apertures, then it may be meaningful to consider their interpretation. The most striking feature of this plot is the relatively slow variation with depth, which presumably means that the energy of the muons occurring in groups is higher than that of unaccompanied muons.

8. Interpretation of underground muon showers

Events in which a number of mesons are observed underground may be due either to the nuclear interaction of muons in the overlaying rock or to the muon component of extensive air showers. The former type of event can be distinguished with most types of apparatus, as a high proportion of the mesons emerge from the rock as pions (Higashi *et al.* 1957) and hence will interact in the apparatus. Also in some cases their divergence can be observed directly since only showers produced close to the tunnel are important, owing to their strong absorption in the rock. Hunter and Trent (1962) have shown that this type of shower is strongly collimated, as would be expected, and makes a negligible contribution to the coincidence rate for screened counter trays separated by more than 3 m. This conclusion is supported by the present experiment, as shown in table 1.

It is important to decide whether the contribution ascribed to extensive air showers is compatible with the known properties of the latter. Unfortunately, the theoretical ideas on air showers are not yet sufficiently developed to enable any critical comparison to be made. However, Greisen (1960) has suggested an empirical expression for the muon structure function which is based on measurements of muon spectra in air showers at sea level by Bennett and Greisen (1961) and supported, at least for showers of large N, by recent measurements of Earnshaw *et al.* (1967). The density of muons of energy greater than E GeV at a distance r metres from the axis of a shower of size N is given by

$$\Delta(N, r, > E) = \frac{14{\cdot}4 r^{-3/4}}{(1+r/320)^{2{\cdot}5}} \left(\frac{N}{10^6}\right)^{3/4} \frac{51}{E+50} \left(\frac{3}{E+2}\right)^{0{\cdot}14 r^{0{\cdot}37}} \quad \mathrm{m}^{-2}$$

$$\text{for } 10 < r < 500.$$

The distribution of shower sizes is also given by Greisen:

$$F(> N) = 3 \times 10^{-8} \left(\frac{N}{10^6}\right)^{-1{\cdot}66 - 0{\cdot}06 \log(N/10^6)} \quad \mathrm{m}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$$

$$\text{for } 10^3 < N < 10^9.$$

(Expressions given by other shower workers are of rather different form but agree broadly with those of Greisen.)

The predicted rate of events in which at least two muons of energy greater than E_1 cross an area of 1 m^2 is then

$$R_2 = \int_N \int_r \{1-(1+\Delta)\,\mathrm{e}^{-\Delta}\} f(N) 2\pi r \, dr \, dN$$

where $f(N)$ can be found from $F(> N)$. This expression has been computed as a function of E_1 and can then be converted to a variation with depth. The latter step requires some assumption about the angular dependence of the showers but, as it is not critically dependent, it is sufficient to take E_1 as 10% greater than the energy required to reach a given depth vertically. The remaining difficulty is the restrictions on the ranges of r and N for which the original formulae are valid. The contribution from events beyond the upper limits would be unimportant but it is obvious that events occurring below the lower limits could be significant. An additional assumption was therefore made that the Greisen function could be extrapolated back to the origin of r. This seemed acceptable as the total number of muons in the region $r < 10$ would only be a small fraction of the total. A similar extrapolation for showers of size $N < 10^3$ seemed more dubious and was not used for the final estimates, although calculations showed that, if the formulae were valid down to $N = 10$, the increase in the predicted rates would be only 25%. The final curve is shown in figure 6. No correction has been applied for the effect of multiple scattering in the rock as the maximum displacement is only about 30 cm. The large spread in the experimental data suggests that the comparison with the computed curve should not be made uncritically, but there is a slight indication that, except near sea level, the rate of pairs is generally higher than predicted. It is possible that the difference is due to the artificial cut-off of showers of $N < 10^3$ or to a steeper rise in the number of muons between 10 and 1 m from the shower axis. Earnshaw *et al.* (1967) have also noted that the Greisen function slightly underestimates the number of muons.

Predictions based on the same model can also be compared with other results from the stack experiment. The ratio $(\mathrm{S}+\mathrm{X}_1+\mathrm{X}_2)/(\mathrm{S}+\mathrm{X}_2)$ is seen from table 1 to have a value of $0{\cdot}24 \pm 0{\cdot}02$, whereas computations using the Greisen function yielded 0·177. This statistic is a particularly useful one because the dependence of stack aperture on the zenith angle distribution is eliminated. The discrepancy is therefore not due to geometrical factors but again suggests that the Greisen function may underestimate the density of muons near the shower axis.

A further extension of the computations gave the muon multiplicity spectrum to be expected in the stack. The theoretical results could be fitted well with a power law of index $-3{\cdot}0$, which is to be compared with the experimental value of $-2{\cdot}6 \pm 0{\cdot}2$. The discrepancy here is probably real, especially as geometrical effects lead to the experimental value being an overestimate of the true value. Examination of the detailed computations showed that, whereas half of all the $S+X_2$ events occurred for shower axes passing more than 18 m from S, the figure for the $S+X_1+X_2$ events was 12 m and for events of high multiplicity ($m \geqslant 5$) 7 m. The increasing dependence on showers at small distances thus accounts for the greater discrepancy if the shower function needs modifying near the axis. The experimental results are not sufficient to determine the modification unambiguously. Some computations were carried out in which, for $r < 10$ m, the term $r^{-0{\cdot}75}$ was replaced by $\sqrt{10}\, r^{-1{\cdot}25}$ (the total number of muons is still finite). This change led to an increase in the twofold rate by a factor of 1·35, the value of $(S+X_1+X_2)/(S+X_2)$ became 0·226 and the multiplicity spectrum $m^{-2{\cdot}4}$. This modification was thus effective in greatly reducing the discrepancies with the experimental results, but undoubtedly a similar end could be achieved by any type of modification which increased the muon density near the axis; the particular form chosen has no physical significance and the very good agreement may well be fortuitous. It should be noted that this experiment only provides information on small transverse momentum transfers in air showers.

In a series of underground experiments the Moscow group (Vernov *et al.* 1962 a, b, 1965) have shown the presence of bundles of muons and have interpreted these as a distinct phenomenon. The Holborn apparatus is of a sufficiently different type that it is hard to compare the results critically, but there does not appear to be any striking conflict. However, the more detailed information obtained on the multiplicity spectrum does not provide any indication of a separate type of event involving many muons. Moreover, studies of muons in air showers at sea level have provided only limited data on the density of muons close to the axis and it has been pointed out by de Beer *et al.* (1966) that errors in core location mean that the density may well have been underestimated in this region. Since there is no further evidence requiring the introduction of special phenomena, as proposed by Vernov, it seems both preferable and allowable to point to the results of this experiment as compatible with a slightly modified form of the muon density distribution deduced by Greisen.

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